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TURBULENCE MODULATION IN TWO-PHASE JETS

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Abstract—A simple model is presented which can be used to predict the modifying effect of a dispersed-phase on the turbulence structure of two-phase jets. It is based on Kolmogorov's concept of spectral energy transfer and takes into account the additional energy dissipation resulting from the inability of dispersed-phase particles to completely follow turbulent eddy fluctuations. According to the analysis presented, high-frequency eddies are attenuated preferentially and a reduction of the total rate of turbulent energy dissipation results. Turbulence intensities were also reduced. Good agreement between predictions and experimental findings were obtained.

1. INTRODUCTION

Two-phase turbulent jets are a commonly occurring type of flow with varied and important technical applications. Combustion of liquid and particulate solid fuels, including rocket propulsion, dusting and spraying for agricultural or forestry purposes, preparation and processing of aerosols are typical examples.

The turbulence structure of the continuous phase was found to be altered as a result of the presence of a dispersed phases in the jet (Goldschmidt & Eskinazi 1966; Abramovich 1970; Hetsroni & Sokolov 1971; Baker *et al.* 1973). This alteration, in turn, affects the rate of jet spreading (Laats & Frishman 1970; Hetsroni & Sokolov 1971), temperature and concentration distributions (Hedman & Smott 1975) as well as the rate of combustion of the dispersed phase (Rudinger 1975).

Hetsroni & Sokolov (1971) conducted an extensive investigation of the effect injected oil droplets have on the turbulence structure of air jets. They found that the presence of the dispersed phase reduces spectral intensity, particularly at higher frequencies; this modulating effect was found to increase with the concentration of the dispersed phase. However, the analysis presented in conjunction with these data predicts these effects only qualitatively and does not offer an explanation for their concentration dependence.

The aim of this paper is to present a model which can be used to estimate the modifying effect of the dispersed-phase on the energy spectrum of continuous-phase turbulent-fluctuations. This model was found to agree reasonably well with experimental data and could thus be used to predict the effect of turbulence modulation on other transport processes.

2. RESPONSE OF DISPERSED-PHASE PARTICLES TO TURBULENT FLUCTUATIONS

The motion of particles suspended in turbulent flows is a topic of great interest whose theoretical aspects have been extensively considered (Tchen 1947; Hinze 1959, 1971; Hjelmfelt & Mokros 1966; Soo 1967). Starting from the Basset–Boussinesq–Oseen solution of the slow motion of a spherical particle in a fluid at rest, Tchen (1947) derived the equation describing the motion of a spherical particle in turbulent flow. This solution was later generalized by Corrsin & Lumley (1956). In its general form, it is a nonlinear partial differential equation; but for non-interacting particles, smaller than the turbulence scale and moving in a locally isotropic, homogeneous and stationary turbulent flow, it reduces to an ordinary differential equation:

$$\frac{\pi D^3}{6}(\rho_d + \chi\rho_c)\frac{\mathrm{d}V}{\mathrm{d}t} = 3\pi\nu\rho_c D(U - V) + \frac{\pi D^3}{6}\rho_c(1+\chi)\frac{\mathrm{d}U}{\mathrm{d}t} + \frac{3D^2\rho_c}{2}\sqrt{(\pi\nu)}\int_{t_0}^{t}\frac{\mathrm{d}U/\mathrm{d}T - \mathrm{d}V/\mathrm{d}T}{\sqrt{(t-T)}}\mathrm{d}T + F, \qquad [1]$$

where D and V are the particle diameter and velocity, t is time, F represents the external field forces acting on the particle, U and ν are the fluid velocity and kinematic viscosity, while ρ_d and ρ_c are the densities of the dispersed and continuous phases respectively.

The coefficient of virtual mass χ accounts for the increased particle inertia when accelerated in a fluid. For spherical particles oscillating in viscous fluids Stokes derived the following expression

$$\chi = 0.5 + 9\sqrt{(\nu/2\omega D^2)} \qquad [2]$$

where ω is the angular frequency.

Rearranging, [1] simplifies to

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \alpha(U - V) + \gamma\frac{\mathrm{d}U}{\mathrm{d}t} + \delta\int_{t_0}^{t}\frac{\mathrm{d}U/\mathrm{d}T - \mathrm{d}V/\mathrm{d}T}{\sqrt{(t-T)}} \qquad [3]$$

where

$$\alpha = \frac{18\nu}{D^2(\phi + \chi)}, \qquad [4]$$

$$\gamma = \frac{1+\chi}{\phi + \chi}, \qquad [5]$$

$$\delta = \frac{9\sqrt{(\nu/\pi D^2)}}{\phi + \chi} \qquad [6]$$

and the density ratio ϕ is ρ_d/ρ_c.

The response of dispersed-phase particles to turbulent fluctuations depends on the ratio between the inertia and viscous forces in [3], which for small Reynolds numbers is given by the dimensionless grouping $[\omega D^2/\nu][(\phi + \chi)/18]$ (Al Taweel & Carley 1971). The vibration number, $N_{Vib} = \omega D^2/\nu$, can thus be considered as a modified form of the Reynolds number which is applicable to periodic flows.

The last term on the R.H.S. of [3] is the history term derived by Basset (1910) and accounts for the additional drag associated with the unsteady motion in viscous fluids. Neglect of this term can result in appreciable error unless the density ratio ϕ is very large (Hjelmfelt & Mokros 1966, El-Tawil 1969), whereas its retention does not result in excessive computational difficulties.

The particle and fluid velocity can be represented by

$$V = v + v', \qquad [7]$$

$$U = u + u', \qquad [8]$$

where the Lagrangian turbulent velocity components u' and v' can be expressed in term of their velocity spectrum.

$$u' = \sum_{n_{\min}}^{n_{\max}} A_n\omega \sin(2\pi nt + \lambda_n), \qquad [9]$$

$$v' = \sum_{n_{\min}}^{n_{\max}} a_n\omega \sin(2\pi nt + \lambda_n - \theta). \qquad [10]$$

Substituting in [3] and solving, the amplitude ratio a_n/A_n and the angle of phase lag θ_n between the fluid and particle fluctuations are given by (Al Taweel & Carley 1971)

$$\theta_n = \tan^{-1}\frac{(\alpha'+\delta')(1+\delta')-(\alpha'+\delta')(\gamma+\delta')}{(\alpha'+\delta')^2+(1+\delta')(\gamma+\delta')}, \qquad [11]$$

$$\frac{a_n}{A_n} = \frac{(\alpha'+\delta')^2+(1+\delta')(\gamma+\delta')}{[(\alpha'+\delta')^2+(1+\delta')^2]\cos\theta_n} \qquad [12]$$

where

$$\alpha' = \alpha/\omega = 18\nu/\omega D^2(\phi+\chi),$$

$$\delta' = \sqrt{(\pi/2\omega)}\delta = 9\sqrt{(\nu/2\omega D^2)}/(\phi+\chi).$$

Due to the deviation of the particle motion from that of the fluid, a fluctuating relative velocity is generated. At any particular frequency in the spectrum, this relative velocity is given by

$$(u_n' - v_n')_{RMS} = \frac{A_n\omega}{\sqrt{(2)}}\sqrt{[1+(a_n/A_n)^2-(2a_n/A_n)\cos\theta_n]} \qquad [13]$$

and the ratio between this relative velocity and the fluid fluctuating velocity becomes

$$R_n = \frac{(u_n'-v_n')_{RMS}}{u'_{n_{RMS}}} = \sqrt{[1+(a_n/A_n)^2-(2a_n/A_n)\cos\theta_n]}. \qquad [14]$$

The variation of the velocity ratio as a function of the vibration number is shown in figure 1 for the cases of oil droplets in air and for air bubbles in water. For the former, relative velocity between the phases is generated because the amplitude of droplet fluctuations is smaller than that of air whereas for the latter the fluctuating motion of the bubbles exceeds that of the water. At small vibration numbers (i.e. small particles, low frequencies or viscous fluids) the particles tend to follow the fluid's turbulent fluctuations, but with increasing vibration numbers the particle motion begins to deviate from that of the fluid. The magnitude of deviation thus depends on both the vibration number and the relative density ϕ.

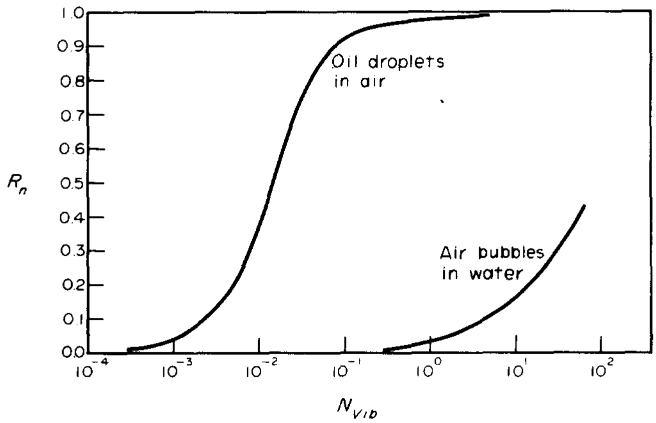

Figure 1. Response of dispersed-phase particles to turbulent fluctuations.

3. TURBULENT ENERGY DISSIPATION BY THE DISPERSED PHASE

From the previous discussion it is evident that dispersed-phase particles generally do not completely follow the turbulent fluctuations of the continuous phase. This gives rise to a fluctuating relative velocity between the phases resulting in dissipation of energy due to the shearing action. This dissipated energy is extracted from the kinetic energy of the fluctuating eddies and thus contributes to the process of turbulent energy. dissipation.

The energy dissipated as a result of the fluctuating relative velocity between the phases is given by

$$\begin{aligned}\text{instantaneous rate of energy} &= \text{drag force} \times \text{relative velocity},\\ \text{dissipation per particle} &= 3\pi\nu\rho_c D(u'_n - v'_n)^2\end{aligned} \qquad [15]$$

and

$$\begin{aligned}\text{average rate of energy} &= 3\pi\nu\rho_c D(u'_n - v'_n)^2_{RMS},\\ \text{dissipation per particle} &= 3\pi\nu\rho_c D u'^2_{nRMS} R_n{}^2.\end{aligned} \qquad [16]$$

Assuming the particles to be uniformly distributed with a concentration of N particles per unit mass of the continuous phase, the average rate of energy dissipation per unit mass of the fluid resulting from the interaction between the phases becomes

$$\begin{aligned}\frac{\mathrm{d}e_n}{\mathrm{d}t} &= 3\pi\nu\rho_c DN u'^2_{nRMS} R_n{}^2,\\ &= \frac{18\,W\nu\rho_c}{D^2\rho_d} u'^2_{nRMS} R_n^2\end{aligned} \qquad [17]$$

where W is the weight concentration of the dispersed phase and e_n is the turbulent energy content per unit mass of the continuous phase having a frequency n.

Snyder & Lumley (1971) and Komasawa *et al.* (1974) in their experimental investigation of particle motion in turbulent flows found, contrary to theoretical reasoning, that there is no significant difference between the non-dimensionalized Lagrangian and Eulerian turbulence spectra when the turbulence is stationary and homogeneous. Thus, by analogy the rate of energy dissipation per unit mass of the fluid and per unit wave number due to the presence of a second phase is given by

$$\frac{18\,W\nu\rho_c}{D^2\kappa\rho_d} u'^2_{\kappa RMS} R_\kappa^2 \qquad [17a]$$

where κ is the wave number.

4. MODULATION OF TURBULENCE SPECTRUM

According to Kolmogorov's concept of spectral energy transfer, energy is extracted from the slow energy-containing eddies and cascades down the spectrum to the fast small-scale eddies which are responsible for the major part of turbulent energy dissipation. At any particular wave number, energy is received from the slower eddies, part of it is dissipated and the remainder is further transfered to eddies of higher wave numbers. Using this concept, Corrsin (1964) and Pao (1965) were able to derive an expression for the turbulent energy spectrum that agreed well with experimental data, particularly those at large Reynolds numbers. An approach similar to theirs will be applied to the case of single and two-phase flows in order to evaluate the turbulence modulating effect of the dispersed-phase.

For frequencies higher than the inertial subrange of the spectrum, the energy flux is given by

$$T(\kappa) = \xi^{-1}\epsilon^{1/3}\kappa^{5/3}E(\kappa) \qquad [18]$$

where $T(\kappa)$ is the energy flux across the wave number κ, ϵ is the rate of turbulent energy dissipation per unit mass of the continuous phase, $E(\kappa)$ is the fraction of turbulent energy associated with κ, and ξ is an empirically determined spectrum constant equal to 1.5. Because of the interaction between the eddies, and the consequent vortex stretch, this energy flux suffers a loss as it cascades across the wave number interval $d\kappa$. For isotropic turbulence it is given by

$$\frac{dT(\kappa)}{d\kappa} = -2\nu\kappa^2 E(\kappa). \quad [19]$$

Substituting from [18] and integrating, the resulting equation is

$$E(\kappa) = \xi\epsilon^{2/3}\kappa^{-5/3}\exp\left[-3/2\xi(\kappa\eta)^{4/3}\right] \quad [20]$$

where η is the Kolmogorov microscale given by

$$\eta = (\nu^3/\epsilon)^{1/4} \quad [21]$$

and

$$\epsilon = 2\nu\int_0^\infty \kappa^2 E(\kappa)\, d\kappa. \quad [22]$$

In the presence of a dispersed phase turbulent energy is dissipated by the interaction between the eddies as well as by the interaction between the phases. Under such conditions, the energy flux $T(\kappa)_{TP}$, the rate of energy dissipation ϵ_{TP} and the fraction of turbulent energy $E(\kappa)_{TP}$ are different from their single-phase values, and [19] is written as

$$\left(\frac{dT}{d\kappa}\right)_{TP} = -2\nu\kappa^2 E(\kappa)_{TP} - \frac{18\,W\nu}{D^2\phi\kappa}u'^2_{\kappa RMS}R_\kappa^2. \quad [23]$$

The L.H.S. of [23] represents the net rate of energy change in the κ^{th} eddy due to the interaction with other eddies and with the dispersed particles. The first term on the R.H.S. refers to the viscous dissipation in the eddies per wave number while the second term represents the energy dissipated, per wave number, as a result of the fluctuating relative velocity generated between the phases. But

$$E(\kappa)_{TP} = u'^2_{\kappa RMS}/2\kappa. \quad [24]$$

Then substituting in [23] and rearranging

$$\frac{dT(\kappa)_{TP}}{d\kappa} = -\left[2\nu\kappa^2 + \frac{36\,W\nu}{D^2\phi}R_\kappa^2\right]E(\kappa)_{TP}. \quad [25]$$

Assuming the same functional form for $T(\kappa)_{TP}$ as in single-phase flow, the energy flux in two-phase systems can be written as

$$T(\kappa)_{TP} = \xi^{-1}\epsilon_{TP}^{1/3}\kappa^{5/3}E(\kappa)_{TP}. \quad [26]$$

Substituting [26] in [25] and integrating

$$E(\kappa)_{TP} = C\kappa^{-5/3}\exp\left[-\frac{3\xi}{2}(\kappa\eta_{TP})^{4/3} - \int\frac{36\xi W\nu R_\kappa^2}{D^2\phi\epsilon_{TP}^{1/3}\kappa^{5/3}}\,d\kappa\right]. \quad [27]$$

The integration constant C is evaluated from the stipulation that at zero dispersed-phase loading ($W = 0$), [27] should reduce to [20]. The modulation of the turbulence spectrum is thus given by

$$\frac{E(\kappa)_{TP}}{E(\kappa)} = \exp\left[-\frac{36\xi W\nu}{D^2\phi\epsilon_{TP}^{1/3}}\int_0^\kappa \frac{R_\kappa^2}{\kappa^{5/3}}\,d\kappa\right]. \quad [28]$$

The turbulence spectrum attenuation ratio at various wave numbers can be easily estimated using [28] since R_κ is an algebraic function of the system properties given by [14]. A single evaluation of the integral in [28] as a function of the upper limit κ is needed; on the other hand, because of the dependence of ϵ_{TP} on $E(\kappa)_{TP}$ an iterative procedure becomes necessary. The value of ϵ_{TP} was obtained from the one-dimensional spectrum by

$$\epsilon_{TP} = \int_0^\infty \left[15\nu\kappa^2 + 36\frac{W\nu}{D^2\phi}R_\kappa^2\right]E(\kappa)_{TP}\,d\kappa. \qquad [29]$$

For two-phase jets, twelve iterations were found to be sufficient for achieving more than 98% convergence.

Baw & Peskin (1971) analysed the modulation of turbulence in gas–solid systems with the assumption that the solid particles do not respond to turbulent fluctuations. This assumption is approximated when fast eddies interact with large particles whose density ratio is very large. The range of applicability of their analysis is thus expected to be limited whereas, because of the absence of such restrictions, the analysis presented in this paper is expected to apply at all wave numbers for gas–solid, liquid–solid, liquid–liquid and liquid–gas systems provided the turbulence can be assumed to be homogeneous and isotropic. If R_κ is set equal to unity, i.e. if it is assumed that the particles do not respond to turbulent fluctuations, [23] reduces to a form very similar to that presented by Baw & Peskin (1971). Furthermore, the present analysis can easily incorporate other factors, such as particle shape, that affect the response of particles to turbulent fluctuation and consequently turbulence modulation.

5. DISCUSSION AND COMPARISON WITH EXPERIMENTAL DATA

Hetsroni & Sokolov (1971) conducted an extensive investigation of two-phase jets in which they introduced $13\,\mu$m droplets of cotton seed oil into axially symmetrical air jets; droplet distribution, turbulence intensity and spectra were measured, among other things. These data offer the opportunity to determine the capability of the proposed model to predict actual situations.

The experimental technique used by Hetsroni & Sokolov for measuring droplet flux resulted in values along the centre line of the jet that are much smaller than those predicted by the accompanying theoretical analysis (equation 19 in their work). On the other hand, good agreement with theoretically predicted trends was obtained by the authors when the fluxes were normalized with respect to the corresponding values at the centre-line of the jet at a distance of 15 nozzle diameters downstream. This discrepancy could be due to the fact that corrections were not incorporated to account for some effects (impaction coefficient, partial hit effect) and also to the initial entrainment taking place near the nozzle. Therefore, for the purpose of this work, dispersed phase loadings at conditions for which turbulence spectra were measured had to be determined by a different approach.

Due to entrainment and jet spreading, as well as to radial dispersion of droplets, centre-line fluxes were found to decrease with the distance from the nozzle. An estimate of the resulting dilution was obtained for the conditions where turbulence spectra were measured by fitting [27] to the value of $E(10)_{TP}$ experimentally obtained at a volumetric flow rate ratio $Q_d/Q_c = 6.06 \times 10^{-6}$. This dilution was then used to calculate the dispersed-phase loading for the other two flow-rate ratios. Using these droplet loadings, the two-phase spectra were computed and normalized such that the area under each curve equaled unity; i.e.

$$\int_0^\infty E(\kappa)_{TP}\,d\kappa = 1 \text{ (figure 2).}$$

As can be seen from figure 2, the agreement between experimentally obtained spectra and

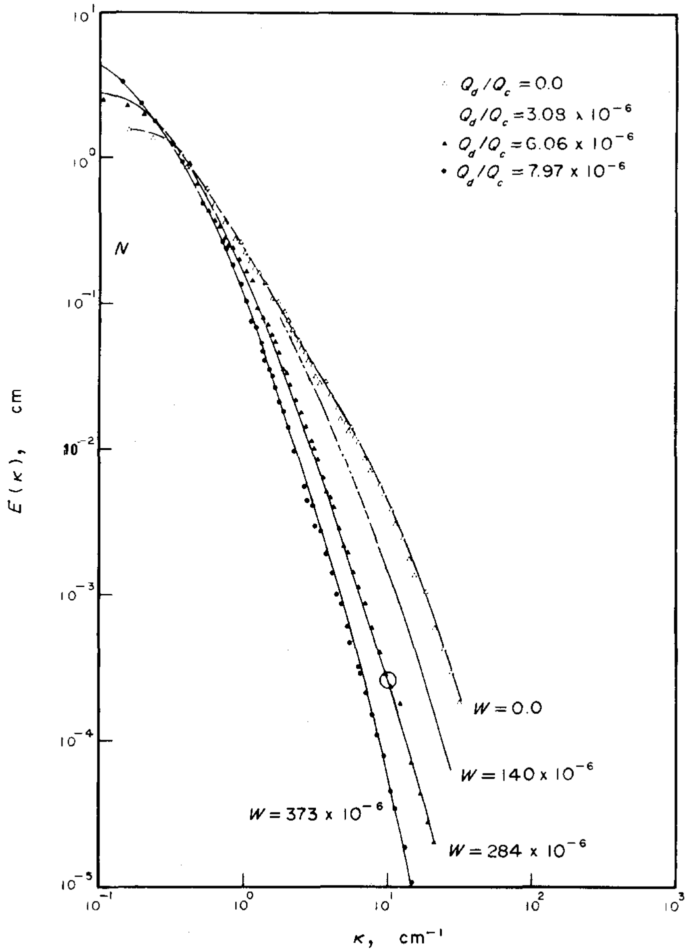

Figure 2. Modulation of turbulence spectra (Encircled point was used for determining dilution ratio). △ ○ ▲ ● Experimental value of Hetsroni & Sokolov (1971). —— Calculated.

those predicted by [28] is very good. This is attributed to the small particles used which were not influenced by the shear flow. The predicted reduction in intensity of high-frequency eddies is in agreement with many experimental and theoretical findings concerning liquid–gas, solid–gas, solid–liquid and gas–liquid systems (Peskin & Rin 1967; Owen 1969; Hetsroni & Einav 1969; Goldschmidt *et al.* 1971; Pirih & Swanson 1972; Boothroyd & Walton 1973).

Several authors have observed that due to the presence of a dispersed-phase, turbulence intensities in two-phase jets can be lower than those of single-phase jets (Hetsroni & Sokolov 1971; Goldschmidt *et al.* 1971; Baker *et al.* 1973). However, based on the observation that energy-containing eddies are largely unaffected by small droplets, Hetsroni & Sokolov considered their turbulence intensity measurements to be almost independent of droplet loading. The results of an examination of the adequacy of this assumption are shown in figure 3. As can be seen from the figure, the three points determined by evaluating the area under the energy spectrum, i.e.

$$\int_0^\infty E(\kappa')_{TP}\, \mathrm{d}\kappa = \overline{u'^2_{TP}},$$

agree very well with the line computed by taking into consideration the attenuation of u'. For these points the value of W was estimated in the manner outlined earlier in the paper. For lack of a better procedure, the values of W for the experimental points obtained at smaller Q_d/Q_c using a *RMS* voltmeter were obtained by assuming them to be proportional to Q_d/Q_c; the droplet

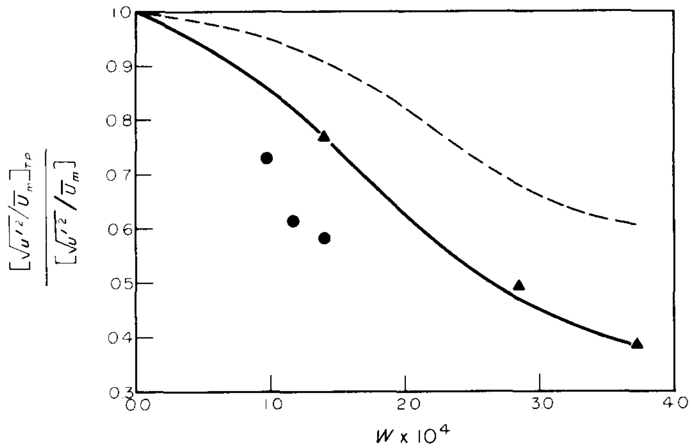

Figure 3. Effect of dispersed-phase loading on the turbulence-intensity reduction ratio. ● $\sqrt{(u'^2)}$ measured using RMS voltmeter, ($u_0 = 50$ m/sec). ▲ $\sqrt{(u'^2)}$ evaluated by integrating the spectra ($u_0 = 61.5$ m/sec). --- computed on the assumption that $\sqrt{(u'^2)}$ is constant. —— computed taking the attenuation of $\sqrt{(u'^2)}$ into consideration.

loadings obtained by fitting [27] to the measured spectra were thus used to calculate the values of W at the smaller Q_d/Q_c ratios. This significant element of uncertainty in the estimate of W, as well as the possible difference in the results of the two methods of measuring $\sqrt{(u'^2_{TP})}$ are the most probable reasons for the poorer fit observed with the second set of data.

A change in the structure of turbulence with a subsequent reduction in turbulence energy level was found necessary to obtain good agreement between the analysis of Danon *et al.* (1976) and the time-average data of Hetsroni & Sokolov (1971), but they did not present a mechanism that can account for such an alteration.

Goldschmidt *et al.* (1971) observed similar reductions in the turbulence intensity of water jets into which nitrogen bubbles were introduced. Unfortunately, the present analysis could not be applied to their data because of insufficient information concerning the bubble size. Furthermore, bubbles introduced in a turbulent water jet have been observed to disintegrate into smaller ones (Sevik & Park 1973); the occurence of such a phenomenon could strongly affect the measured spectra.

When a dispersed phase is introduced into a jet, the fluctuations of the high-frequency energy-dissipating eddies are dampened, and this results in a reduction of the rate of turbulent energy dissipation within the continuous phase. On the other hand, additional turbulent energy is dissipated as a consequence of the fluctuating relative motion between the phases. The total rate of energy dissipation thus becomes the sum of the turbulent energy dissipated by the eddies interacting amongst themselves and that resulting from the interaction between the eddies and the particles of the dispersed phase. This is analytically expressed by [29] where the first term on the R.H.S. represents eddy–eddy interaction, while the second term represents particle–eddy interaction.

The effect of dispersed-phase loading on the total rate of turbulent energy dissipation is shown in figure 4 for various particle sizes. Smaller particles are more effective reducers of turbulent energy dissipation in air jets because they provide a larger surface area on which shear can take place while still generating sufficient fluctuating velocity relative to air eddies. Care must be exercised, however, in extending this conclusion to other systems without due consideration to the variation in turbulence spectrum and particle response to turbulent fluctuations.

The reduced turbulence dissipation is expected to supress turbulence generation, a concept which can be used to explain the phenomenon of drag reduction observed in pipe flow of dilute solid suspensions (Al Taweel & Landau).

The implications of both the absolute magnitude and relative shape of the two-phase energy

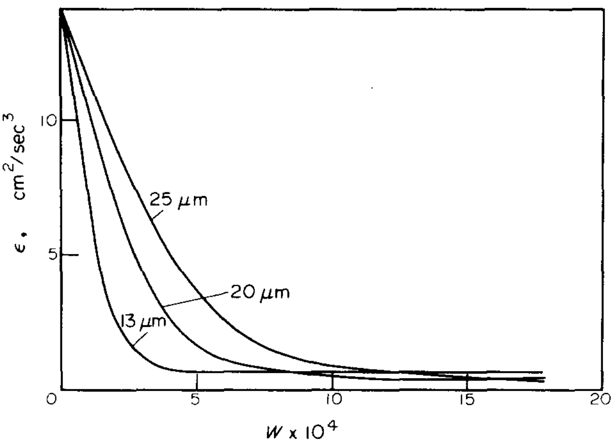

Figure 4. Effect of dispersed-phase loading on the total rate of turbulent dissipation.

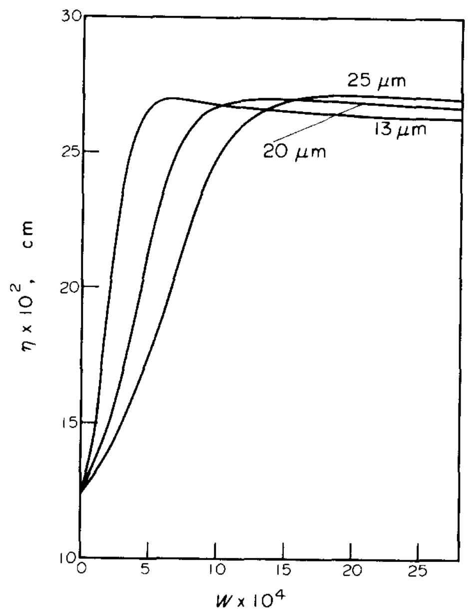

Figure 5. Variation of turbulence microscale in two-phase jets.

spectra can be seen in the characteristic turbulence scales. The reduced rate of turbulent energy dissipation is thus reflected in the variation of the Kolmogorov microscale η in two-phase jets (figure 5). This enlargement of the turbulence microscale could have strong effects on some transfer processes that are controlled by them (Bremhorst & Bullock 1973); thus for instance it has been found that heat-transfer coefficients for air flowing in pipes have been reduced upon the addition of small amounts of fine solids (Kane & Pfeffer 1973).

6. CONCLUSIONS

The introduction of a dispersed phase into single-phase jets was found to alter the turbulence structure of the continuous phase. This is attributed to the inability of dispersed-phase particles to completely follow the fluctuating motion of turbulent eddies.

Applying Kolmogorov's concept of spectral energy transfer and taking into account the additional dissipation arising from the presence of a dispersed phase, an expression for the turbulence spectrum attenuation ratio was derived assuming isotropic and homogeneous turbulence. Predictions of this analysis were found to agree reasonably well with experimental data obtained for oil droplets dispersed into air jets.

It was found that the presence of a dispersed-phase alters the turbulence structure of jets in the following manner,

(a) Attenuation of the high-frequency fluctuations with small alteration of the energy-containing low-frequency eddies. The magnitude of this damping increases with dispersed-phase loading and decreases with particle diameter.

(b) Turbulence intensities within the continuous phase are reduced as a result of the presence of the dispersed phase.

(c) The presence of a dispersed phase results in the additional dissipation of turbulent energy. However, due to the reduction in dissipation within the continuous phase, the total rate of energy dissipation in two-phase jets is reduced; as a result of this Kolmogorov's microscale of turbulence increases.

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